QUIZ 17 SOLUTIONS: LESSONS 22-23
OCTOBER 26, 2018

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [4 pts] Find $\dfrac{dz}{dt}$ given

$$z = x^2y^2, \quad x = \sin t, \quad y = 7t^4.$$

Write your answer in terms of x, y, t.

Solution: We use the chain rule for multivariable functions:

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}.$$

We find our four pieces:

$$\begin{aligned}
\frac{\partial z}{\partial x} &= \frac{\partial}{\partial x}(x^2y^2) \\
&= y^2\left[\frac{\partial}{\partial x}x^2\right] \\
&= y^2(2x) = 2xy^2 \\
\frac{\partial z}{\partial y} &= \frac{\partial}{\partial y}(x^2y^2) \\
&= x^2\left[\frac{\partial}{\partial y}(y^2)\right] \\
&= x^2(2y) = 2x^2y \\
\frac{dx}{dt} &= \cos t \\
\frac{dy}{dt} &= 28y^3
\end{aligned}$$

Thus,

$$\boxed{\frac{dz}{dt} = (2xy^2)(\cos t) + (2x^2y)(7t^4)}.$$

2. Let

$$g(x,y) = \frac{1}{2}x^2 + xy + \frac{9}{4}y^4 - 2.$$

(a) [3 pts] Find all the critical points of $g(x,y)$.

Solution: We find the critical points. Differentiating, we get

$$g_x = x + y \quad \text{and} \quad g_y = x + 9y^3.$$

We set these equal to zero and solve for (x,y). Write

$$0 = g_x = x + y \quad \Rightarrow \quad x = -y$$

and

$$0 = g_y = x + 9y^3 \quad \Rightarrow \quad 0 = -y + 9y^3 \quad \Rightarrow \quad 0 = y(-1 + 9y^2).$$

Then, either $y = 0$ or $-1 + 9y^2 = 0$. If $-1 + 9y^2 = 0$, then

$$9y^2 = 1 \quad \Rightarrow \quad y^2 = \frac{1}{9}.$$

Hence, $y = \pm\frac{1}{3}$. Putting this together, we see that our critical points are

$$\boxed{\left(\frac{1}{3}, -\frac{1}{3}\right) \quad \text{and} \quad \left(-\frac{1}{3}, \frac{1}{3}\right)}.$$

(b) [1 pt] Write down the discriminant of $g(x,y)$.

Solution: The discriminant of $g(x,y)$ is given by

$$D(x,y) = g_{xx}g_{yy} - (g_{xy})^2.$$

Since

$$g_{xx} = 1, \quad g_{yy} = 27y^2, \quad g_{xy} = 1,$$

we see

$$D(x,y) = (1)(27y^2) - (1)^2 = \boxed{27y^2 - 1}.$$

(c) [2 pts] Classify all the critical points of $g(x,y)$.

Solution: We need to check our two critical points. Write

$$\begin{aligned} D\left(\frac{1}{3}, -\frac{1}{3}\right) &= 27\left(-\frac{1}{3}\right)^2 - 1 \\ &= 27\left(\frac{1}{9}\right) - 1 \\ &= 3 - 1 = 2 > 0 \end{aligned}$$

$$\begin{aligned} D\left(-\frac{1}{3}, \frac{1}{3}\right) &= 27\left(\frac{1}{3}\right)^2 - 1 \\ &= 27\left(\frac{1}{9}\right) - 1 \\ &= 3 - 1 = 2 > 0 \end{aligned}$$

Critical Point	$D(x_0, y_0)$	$g_{xx}(x_0, y_0)$	Classification
$\left(\frac{1}{3}, -\frac{1}{3}\right)$	$2 > 0$	$1 > 0$	local min
$\left(-\frac{1}{3}, \frac{1}{3}\right)$	$2 > 0$	$1 > 0$	local min